ECE 307
Spring 2018
Prof. George Gross
Room 4052 ECEB

Homework 4
Date due: Friday, February 16, 2018

1. **Place** each of the following *LP*s into *standard form* and **define** the corresponding $\underline{\boldsymbol{A}}$, $\underline{\boldsymbol{b}}$ and $\underline{\boldsymbol{c}}$ arrays

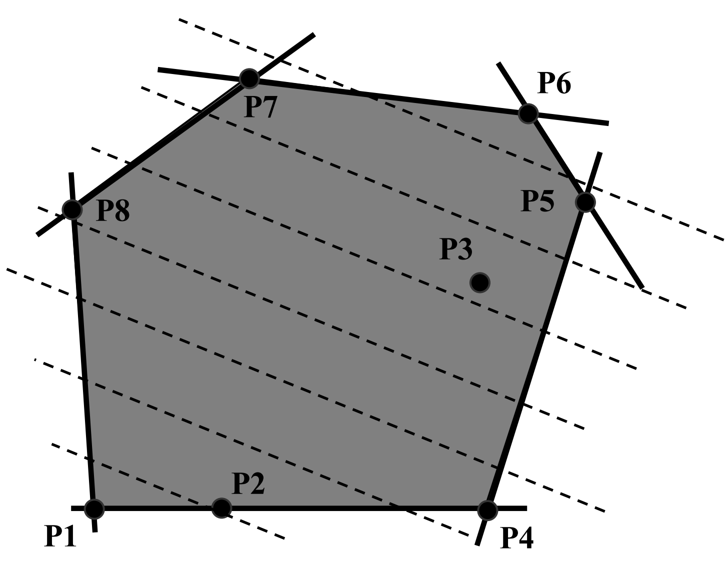
(a)

$$
\begin{aligned}
&\max \quad 45x_1 + 15x_3 \\
&s.t. \\
&4x_1 - 2x_2 + 9x_3 = 22 \\
&-2x_1 + 5x_2 - x_3 \geq 1 \\
&x_1 - x_2 \leq 5 \\
&x_1,\ x_2,\ x_3 \geq 0
\end{aligned}
$$

(b)

$$
\begin{aligned}
&\max \quad 15(x_1 + 2x_2) + 11(x_2 - x_3) \\
&s.t. \\
&\quad 3x_1 \geq x_1 + x_2 + x_3 \\
&\quad 0 \leq x_j \leq 3 \qquad j = 1,2,3
\end{aligned}
$$

2. The following plot shows several feasible points in a linear program and contours of its objective function.



(a) **Solve** the problem graphically

(b) **Determine** whether the following sequence of solutions P1, P7, and P6 could have been generated by the application of the simplex algorithm to the corresponding *LP* problem in standard form. **Provide** the rationale for your answer.

3. Problem 2.30 in **Ravindran**, p.67

4. Problem 2.31 in **Ravindran**, p.68

5. Problem 2.32 in **Ravindran**, p.68

6. Consider the linear program

$$
\begin{aligned}
max \quad & 4y_1 + 5y_2 \\
s.t. \quad & \\
& -y_1 + y_2 \leq 4 \\
& y_1 - y_2 \leq 10 \\
& y_1,\ y_2 \geq 0
\end{aligned}
$$

(a) **Show** graphically that the model is unbounded.

(b) Add slacks y_3 and y_4 to **place** the model in standard form.

(c) Starting with an all slacks basic feasible solution, **apply** simplex algorithm to establish that the original model is unbounded.

7. Consider the *LP*

$$
\begin{aligned}
max \quad & x_1 \\
s.t. \quad & \\
& 6x_1 + 3x_2 \leq 18 \\
& 12x_1 - 3x_2 \leq 0 \\
& x_1,\ x_2 \geq 0
\end{aligned}
$$

(a) **Solve** the problem graphically.

(b) Add slack variables to **place** the model in standard form.

(c) **Apply** the simplex algorithm to compute an optimal solution starting with an all-slack-variables *basic feasible solution*.

8. The NCAA is making plans for distributing tickets to the upcoming regional basketball championships. The up to 10,000 available seats will be divided between the media, the competing universities, and the general public. Media people are admitted free, but the NCAA receives 45 *$* per ticket from universities and 100 *$* per ticket from the general public. At least 500 tickets must be reserved for the media, and at least half as many tickets should go to the competing universities as to the general public. Within these restrictions, the NCAA wishes to find the allocation that raises the most money. An optimization is given in the table.

solution variable (*max*) = *$* 775,833.333								
variable sensitivity analysis								
Name	optimal value	basic non-basic	lower bound	upper bound	objective coefficient	reduced objective	lower range	upper range
x_1	500.00	basic	0.000	¥	0.000	0.000	- ¥	81.667
x_2	3166.667	basic	0.000	¥	45.000	0.000	-200.000	100.00
x_3	6333.333	basic	0.000	¥	100.000	0.000	45.000	¥
constraint sensitivity analysis								
name	type	optimal dual	r.h.s. coefficient	slack	lower range	upper range		
seats	$\leq$	81.667	10000.000	0.000	500.000	¥		
other	$\geq$	-36.667	0.000	-0.000	-4750.000	9500.00		
media	$\geq$	-81.667	500.000	-0.000	0.000	10000.000		

(a) **Formulate** this problem as an *LP*. **Explain** each decision variable, **identify** the coefficients in the objective function and **state** each constraint.

(b) **Interpret** the left-hand-side coefficients of the decision variables in the constraints

9. Consider the statement in the problem 8 above. **Answer** the following questions

(a) What is the marginal cost to the NCAA of each seat guaranteed for the media

(b) Suppose that there is an alternative arrangement of the dome where the games will be played that can provide 15,000 seats. How much additional revenue would be gained from the expanded seating? How much would it be for 20,000 seats?

(c) Since television revenue provides most of the income for NCAA events, another proposal would reduce the price of general public tickets to \$ 50. How much revenue would be lost from this change? What if the price were \$ 30 ?

(d) Media-hating coach Sobby Day wants the NCAA to restrict media seats to 20 % of those allocated for universities. Could this policy change the optimal solution? How about 10 %?

(e) To accommodate high demand from student supporters of participating universities, the NCAA is considering marketing a new "scrunch seat" that consumes only 80 % of a regular bleacher seat but counts fully against the "university $\geq$ half public" rule. Could an optimal solution allocate any such seats at a ticket price of \$ 35? At a price of \$ 25?

10. Professor Proof is trying to arrange for the implementation in a computer program of his latest operations research algorithm. He can contract with any mix of three sources for help: unlimited hours from undergraduates at 4 *$/hour*; up to 500 *hours* of graduate students at the rate of 10 *$/hour*, or unlimited help from professional programmers at 25 *$/hour*. The full project would take a professional at least 1000 *hours* but graduate students are only 0.3 as productive, and undergraduates, 0.2. Proof only has 164 *hours* of his own time to devote to the effort, and he knows from experience that undergraduate programmers require more supervision than graduate

solution variable (*min*) = 24,917.647								
variable sensitivity analysis								
name	optimal value	basic/ non-basic	lower bound	upper bound	objective coefficient	reduced objective	lower range	upper range
x_1	82.353	basic	0.000	∞	4.000	0.000	-4.444	5.000
x_2	0.000	non-basic	0.000	5000	10.000	2.235	7.765	∞
x_3	983.529	basic	0.000	∞	25.000	0.000	20.000	31.333
constraint sensitivity analysis								
name	type	optimal dual	rhs coefficient	slack	lower range	upper range		
professional	$\geq$	25.882	1000.000	-0.000	164.000	1093.333		
proof	$\leq$	-5.882	164.000	0.000	150.000	1000.00		
graduate	$\leq$	0.000	500.000	500.000	0.000	∞		

students, who, in turn, require more than professionals. In particular, he estimates that he will have to invest 0.2 *hours* of his own time per hour of undergraduate programming, 0.1 *hour* of his time per hour of graduate programming and 0.05 *hour* of his time *per hour* of professional programming. The table above summarizes the optimization output.

(a) Briefly **explain** how this problem can be modeled by the following *LP*

$$\begin{array}{ll} min & 4x_1 + 10x_2 + 25x_3 \\ s.t. & \\ & 0.2x_1 + 0.3x_2 + x_3 \geq 1{,}000 \\ & 0.2x_1 + 0.1x_2 + 0.05x_3 \leq 164 \\ & x_2 \leq 500 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

(b) **Identify** the decision variables, **state** the objective function and **formulate** each constraint.

(c) **Interpret** the left-hand-side coefficients of each decision variable in part *(a)* as inputs and outputs of resources per unit activity.

11. Problem 4.23 in **Ravindran**, p.214

12. Problem 4.13*(a)* in **Ravindran**, p.210

13. Problem 4.15 in **Ravindran**, p.211

14. Problem 4.16 in **Ravindran**, p.211

15. Problem 4.17 in **Ravindran**, p.211

16. For each of the *LP*s in *(a) - (d)*

(i) **state** the corresponding dual problem of the primal as given

(ii) **state** the two *sets* of complimentary slackness conditions.

(a)

$$\begin{array}{ll} max & 44x_1 - 3x_2 + 15x_3 + 56x_4 \\ s.t. & \\ & x_1 + x_2 + x_3 + x_4 = 20 \\ & x_1 - x_2 \leq 0 \\ & 9x_1 - 3x_2 + x_3 - x_4 \leq 164 \\ & x_1, \dots, x_4 \geq 0 \end{array}$$

(b)

$$
\begin{array}{ll}
\min & 5x_1 + x_2 - 4x_3 \\
s.t. & \\
& x_1 + x_2 + x_3 + x_4 = 19 \\
& 4x_2 + 8x_4 \le 55 \\
& x_1 + 6x_2 - x_3 \ge 7 \\
& x_2, x_3 \ge 0, \quad x_4 \le 0
\end{array}
$$

(c)

$$
\begin{array}{ll}
\max & 19y_1 + 4y_2 - 8z_2 \\
s.t. & \\
& 11y_1 + y_2 + z_1 = 15 \\
& z_1 + 5z_2 \le 0 \\
& y_1 - y_2 + z_2 \ge 4 \\
& y_1, y_2 \ge 0
\end{array}
$$

(d)

$$
\begin{array}{ll}
\max & 10(x_3 + x_4) \\
s.t. & \\
& \sum_{j=1}^{4} x_j = 80 \\
& x_j - 2x_{j+1} \ge 0 \quad j = 1,2,3 \\
& x_1, x_2 \ge 0 \qquad x_3, x_4 \quad \textit{unspecified}
\end{array}
$$